19.

$(1,1)$ $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$

find the curve - find y

$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-\frac{1}{2}}$

$y = \frac{1}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$y = \sqrt{x} + c$

initial conditions

$1 = \sqrt{1} + c$ $c = 0$

$y = \sqrt{x}$

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13.

$x = \frac{y^3}{3} + \frac{1}{4y}$ $y=1$ $y=3$

$\frac{dx}{dy} = y^2 - \frac{1}{4y^2}$

$\int_1^3 \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2}\, dy$

$\int_1^3 \sqrt{1 + y^4 - \frac{2}{4} + \frac{1}{16y^4}}\, dy$

$\int_1^3 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}\, dy$

$\int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy$

$\int_1^3 y^2 + \frac{1}{4y^2}\, dy$

$\frac{y^3}{3} - \frac{1}{4y} \Big|_1^3$

$\left(\frac{3^3}{3} - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$

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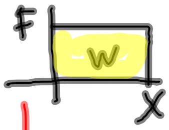
7.5 Work

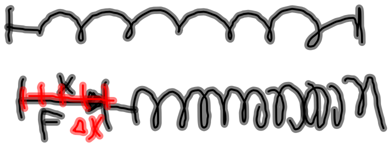
If force is constant, $W = F \cdot x$ (force · distance)



If force varies, $W = \int_a^b F\,dx$ — function of x

why?

rest length



compress

$$\Delta W = F_i \Delta x$$

$$W \approx \sum F_i \Delta x$$

$$W = \lim_{\Delta x \to 0} \sum F_i \Delta x$$

$$= \int_a^b F\,dx$$

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Hooke's law for springs: $F = kx$

If 50 N compresses a spring .2 m. How much work is done?

1. find k

$$50 = k(.2)$$

$$k = \frac{50}{.2} = 250$$

2. work

$$\int_0^{.2} 250x\,dx$$

$$= 5\ N \cdot m = 5\ J$$

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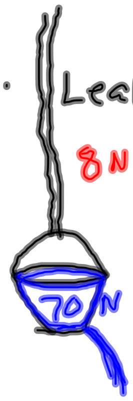
Ex 2. Leaky bucket



8N of rope

20 m

leaks at constant rate

$y = m(x - x_1) + y_1$

rope density $= .4 \frac{N}{m}$

$.4 \frac{N}{m} \cdot 20m = 8N$

bucket: 22 N

Work to lift bucket $= 22N \cdot 20m = 440\,J$

water: $F(x)$ is linear

$x=0 \quad F=70$

$x=20 \quad F=0$

$$F(x) = \frac{0-70}{20-0}(x-0)+70$$

$$\int_0^{20} -\frac{7}{2}x + 70\,dx = 700\,J$$

rope

$$\int_0^{20} -\frac{2}{5}x + 8\,dx = 80\,J$$

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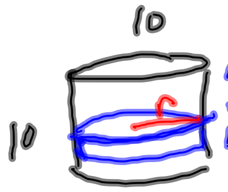
pump water out of a tank

density of water

$62.4 \frac{lb}{ft^3}$



y = distance

$r = 5$

full. pump over the top

ΔW = work for 1 slice

$= F \cdot y$ - distance

$\approx$ weight $\cdot y$

$= 62.4 \frac{lb}{\cancel{ft^3}} \cdot \Delta V \cancel{ft^3} \cdot y\ ft$

$$W \approx \sum 62.4\, \Delta V \cdot y$$

$$\Delta V = \pi r^2 \Delta y$$

$$W \approx \sum 62.4\, \pi \cdot 5^2 \cdot y\, \Delta y$$

$$W = \int_0^{10} 62.4\pi \cdot 5^2\, y\, dy$$

$$= 245{,}044 \text{ ft-lbs}$$

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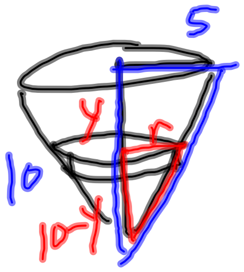

$$\int_0^{10} 62.4\,\pi\left(\tfrac{1}{2}(10-y)\right)^2 y\;dy$$

similar Δ's

$$\frac{5}{10} = \frac{r}{10-y}$$

$$r = \frac{5}{10}(10-y)$$

Ex 3   2 ft from top

olive oil

$$\int_2^{10} 57\,\pi\left(\tfrac{1}{2}(10-y)\right)^2 y\,dy$$

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